GR4J - SRG

The GR4J model is a catchment water balance model that relates runoff to rainfall and evapotranspiration using daily data. The model contains two stores and has four parameters.
Scale

GR4J operates at a catchment scale with a daily time-step.
The development of the GR4J model was initiated by Claude Michel at the beginning of the 1980s at Cemagref, a public research institute in France. The first version of the model only had a single parameter. Further development of the GR4J model was undertaken using a modelling approach where large numbers of catchments were used to evaluate and improve the model.

The GR4J modelling approach is mainly empirical (Michel et al., 2006), and consists of searching data for the most efficient model structures, with the objective of getting a general, efficient and robust model. The result being a parsimonious hydrological model, with successive improved versions. The main stages of the GR4J model development were:

- 3-parameter version proposed by Edijatno and Michel (1989) and Edijatno (1991). This provided the groundwork for further model development through testing and refinement;
- 4-parameter version proposed by Nascimento (1995) and detailed by Edijatno et al. (1999) (with one fixed parameter);
- 4-parameter version proposed by Perrin (2000) and detailed by Perrin (2002) and Perrin et al. (2003);
- 5-parameter version proposed by Le Moine (2008).
Scientific provenance

The successive versions of GR4J were widely tested on large sets of catchments in France but also in other countries, using demanding testing frameworks (Andréassian et al., 2009). The GR4J model has also been compared with other hydrological models and has provided comparatively good results (see eg Perrin et al., 2001; 2003; Vaze et al., 2011).
Version

Source v3.8.10
Dependencies

None.
Structure and processes

The mathematical details provided below follow the presentation of the model made by Perrin et al. (2003). Figure 1 shows a schematic diagram of the model.

Figure 1. Schematic diagram of the GR4J model
In the following, for calculations at a given time-step, we note $P$ the rainfall depth and $E$ the potential evapotranspiration estimate that are inputs to the model. $P$ is an estimate of the areal catchment rainfall that can be computed by any interpolation method from available rain gauges. $E$ can be based on long-term average monthly or daily values, which means the same potential evapotranspiration series could be repeated every year, although a recorded time series of $E$ would be expected to give a better result.

All water quantities (input, output, internal variables) are expressed in mm, by dividing water volumes by catchment area, when necessary. All the operations described below are relative to a given time-step and correspond to a discrete model formulation (obtained after integration of the continuous formulation over the time-step).

**Determination of net rainfall and PE**

The first operation is the subtraction of $E$ from $P$ to determine either a net rainfall $P_n$ or a net evapotranspiration capacity $E_n$. In GR4J, this operation is computed as if there were an interception storage of zero capacity. $P_n$ and $E_n$ are computed with the following equations:

**Equation 1**

\[
\text{if } (P \geq E) \text{ then } (P_n = P - E) \text{ and } (E_n = 0)
\]

**Equation 2**

\[(P_n = 0) \text{ and } (E_n = E - P)\]

**Production store**

This store can be considered as a soil moisture accounting (SMA) store. In case $P_n$ is not zero, a part $P_s$ of $P_n$ fills the production store. It is determined as a function of the level $S$ in the store by:

**Equation 3**

\[
P_s = \frac{P_n}{1 + \frac{S}{x_1} \cdot \tanh\left(\frac{P_n}{x_1}\right)}
\]

\[
= x_1 \cdot \left(1 - \left(\frac{S}{x_1}\right)^2\right) \cdot \tanh\left(\frac{P_n}{x_1}\right)
\]

where the terms are defined in Table 1.

<table>
<thead>
<tr>
<th><strong>Table 1. Model parameter definitions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$E$</td>
</tr>
<tr>
<td>$E_n$</td>
</tr>
<tr>
<td>$E_s$</td>
</tr>
<tr>
<td>$F(x_2)$</td>
</tr>
<tr>
<td>$P$</td>
</tr>
<tr>
<td>Symbol</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Perc</td>
</tr>
<tr>
<td>( P_n )</td>
</tr>
<tr>
<td>( P_r )</td>
</tr>
<tr>
<td>( P_n - P_s )</td>
</tr>
<tr>
<td>( P_s )</td>
</tr>
<tr>
<td>( Q )</td>
</tr>
<tr>
<td>( Q1 )</td>
</tr>
<tr>
<td>( Q9 )</td>
</tr>
<tr>
<td>( Q_d )</td>
</tr>
<tr>
<td>( Q_r )</td>
</tr>
<tr>
<td>( R )</td>
</tr>
<tr>
<td>( S )</td>
</tr>
<tr>
<td>( UH1, UH2 )</td>
</tr>
<tr>
<td>( x_1 )</td>
</tr>
<tr>
<td>( x_2 )</td>
</tr>
<tr>
<td>( x_3 )</td>
</tr>
<tr>
<td>( x_4 )</td>
</tr>
</tbody>
</table>

Equation 3 and Equation 4 result from the integration over the time-step of the differential equations that have a parabolic form with terms in \((S/x_1)^3\), as detailed by (Edijatno and Michel, 1989).

In the other case, when \( E_n \) is not zero, an actual evaporation rate is determined as a function of the level in the production store to calculate the quantity \( Es \) of water that will evaporate from the store. It is obtained by:

\[
Es = \frac{S(2 - \frac{S}{x_1}).\tanh\left(\frac{En}{x_1}\right)}{1 + (1 - \frac{S}{x_1}).\tanh\left(\frac{En}{x_1}\right)}
\]

The water content in the production store is then updated with:

\[
S = S - Es + Ps
\]

Note that \( S \) can never exceed \( x_1 \). A representation of the rating curves obtained with Equation 3 and Equation 4 is shown in Figure 2.

Figure 2. Behaviour of the production functions \((E_n/E_s; \text{solid line}; P_n/P_s; \text{dashed line})\) as a function of storage rate \(S/x_1\) for different values of \(E_n/x_1\) or \(P_n/x_1\).
A percolation leakage $P_{rc}$ from the production store is then calculated as a power function of the reservoir content:

Equation 6

$$P_{rc} = S \left( 1 - \left[ 1 + \left( \frac{4}{9} \frac{S}{x_1} \right)^4 \right]^{-1/4} \right)$$

$P_{rc}$ is always lower than $S$. The reservoir content becomes:

Equation 7

$$S = S - P_{rc}$$

The percolation function in Equation 6 occurs as if it originated from a store with a maximum capacity of $9 - 4 \cdot x_1$. Given the power law of the mathematical formulation, this means that the percolation does not contribute much to the stream flow and is interesting mainly for low flow simulation.

**Linear routing with unit hygrographs**

The total quantity $P_r$ of water that reaches the routing functions is given by:

Equation 8

$$P_r = P_{rc} + (P_n - P_s)$$
$P_r$ is divided into two flow components according to a fixed split: 90% of $P_r$ is routed by a unit hydrograph $UH1$ and then a non-linear routing store, and the remaining 10% of $P_r$ is routed by a single unit hydrograph $UH2$. With $UH1$ and $UH2$, one can simulate the time lag between the rainfall event and the resulting stream flow peak. Their ordinates are used in the model to spread effective rainfall over several successive time-steps. Both unit hydrographs depend on the same time parameter $x_4$ expressed in days. However, $UH1$ has a time base of $x_4$ days whereas $UH2$ has a time base of $2x_4$ days. $x_4$ can take real values and is greater than 0.5 days.

In their discrete form, unit hydrographs $UH1$ and $UH2$ have $n$ and $m$ ordinates respectively, where $n$ and $m$ are the smallest integers exceeding $x_4$ and $2x_4$ respectively. This means that the water is staggered into $n$ unit hydrograph inputs for $UH1$ and $m$ inputs for $UH2$.

The ordinates of both unit hydrographs are derived from the corresponding S-curves (cumulative proportion of the input with time) denoted by $SH1$ and $SH2$ respectively. $SH1$ is defined along time $t$ by:

\begin{align*}
\text{Equation 9} & \quad \text{For } t \leq 0, \quad SH1(t) = 0 \\
\text{Equation 10} & \quad \text{For } 0 < t < x_4, \quad SH1(t) = \left(\frac{t}{x_4}\right)^{\frac{5}{2}} \\
\text{Equation 11} & \quad \text{For } t \geq x_4, \quad SH1(t) = 1
\end{align*}

$SH2$ is similarly defined by:

\begin{align*}
\text{Equation 12} & \quad \text{For } t \leq 0, \quad SH2(t) = 0 \\
\text{Equation 13} & \quad \text{For } 0 < t \leq x_4, \quad SH2(t) = \frac{1}{2} \left(\frac{t}{x_4}\right)^{\frac{5}{2}} \\
\text{Equation 14} & \quad \text{For } x_4 < t < 2 \cdot x_4, \quad SH2(t) = 1 - \frac{1}{2} \left(2 - \frac{t}{x_4}\right)^{\frac{5}{2}} \\
\text{Equation 15} & \quad \text{For } t \geq 2 \cdot x_4, \quad SH2(t) = 1
\end{align*}

$UH1$ and $UH2$ ordinates are then calculated by:

\begin{align*}
\text{Equation 16} & \quad UH1(j) = SH1(j) - SH1(j - 1) \\
\text{Equation 17} & \quad UH2(j) = SH2(j) - SH2(j - 1)
\end{align*}
where:

\[ j \text{ is an integer.} \]

If 0.5 \( x_4 \) \( \leq 1 \), \( UH1 \) has a single ordinate equal to one and \( UH2 \) has only two ordinates. Figure 3 shows an example of unit hydrograph ordinates for \( x_4 = 3.8 \) days.

Figure 3. Example of the ordinates of \( UH1 \) and \( UH2 \) for parameter \( x_4 = 3.8 \) days

At each time-step, the outputs \( Q9 \) and \( Q1 \) of the two unit hydrographs correspond to the discrete convolution products and are given by:

\[
Q9(k) = 0.9 \sum_{j=1}^{1} UH1(j) \cdot \text{Pr}(k - j + 1)
\]

**Equation 18**

\[
Q1(k) = 0.1 \sum_{j=1}^{m} UH2(j) \cdot \text{Pr}(k - j + 1)
\]

**Equation 19**

where:

\[
\begin{align*}
  l &= \text{int}(x_4) + 1; \\
  m &= \text{int}(2 \cdot x_4) + 1
\end{align*}
\]

**Equation 20**
Inter catchment groundwater exchange

A groundwater exchange term $F$ that acts on both flow components, is then calculated as:

\[
F = x_2 \left( \frac{R}{x_3} \right)^2
\]

Where $R$ is the level in the routing store, $x_3$ its “reference” capacity and $x_2$ the water exchange coefficient. $x_2$ can be either positive in case of water imports, negative for water exports or zero when there is no water exchange. The higher the level in the routing store, the larger the exchange. In absolute value, $F$ cannot be greater than $x_2$; $x_2$ represents the maximum quantity of water that can be added (or released) to (from) each model flow component when the routing store level equals $x_3$. Note that Le Moine (2008) proposed an improved formulation of this function, with an additional parameter.

Non linear routing store

The level in the routing store is updated by adding the output $Q_{9}$ of $UH1$ and $F$ as follows:

\[
R = \max(0; R + Q_{9} + F)
\]

The outflow $Q_r$ of the reservoir is then calculated as:

\[
Q_r = R \left( 1 - \left( 1 + \left( \frac{R}{x_3} \right)^4 \right)^{-\frac{1}{4}} \right)
\]

$Q_r$ is always lower than $R$, as shown in Figure 4. The formulation of the output of the store is the same as the percolation from the SMA store. The level in the reservoir becomes:

\[
R = R - Q_r
\]

Note that, although the reservoir can receive a water input greater than the saturation deficit $x_3\cdot R$ at the beginning of a time-step, the level in the reservoir can never exceed the capacity $x_3$ at the end of a time-step, as shown in Figure 4. Therefore, the capacity $x_3$ could be called the "one day ahead maximum capacity". This routing store is able to simulate long stream flow recessions, when necessary.

Figure 4. Illustration of the outflow $Q_r$ from the routing reservoir as a function of the level in the store after the introduction of input $Q_{9}$
Total stream flow

Like the content of the routing store, the output $Q1$ of $UH2$ is subject to the same water exchange $F$ to give the flow component $Q_d$ as follows:

Equation 25

$$Q_d = \max(0; Q1 + F)$$

Total stream flow $Q$ is finally obtained by:

Equation 26

$$Q = Qr + Qd$$

The Source implementation of GR4J includes a baseflow filter than can be used to estimate a baseflow amount from the overall runoff flux, $Q$. GR4J uses the same baseflow filter as is used in the observed catchment runoff depth model (see the Observed catchment runoff depth - SRG page for more information).
Input data

The model requires daily rainfall and potential evapotranspiration data. The rainfall and evaporation data sets need to be continuous and overlapping.
Parameters or settings

Information on parameters is provided in Table 2. All four parameters are real numbers. \( x_1 \) and \( x_3 \) are positive, \( x_4 \) is greater than 0.5 and \( x_2 \) can be either positive zero or negative.

Table 2: Parameters in GR4J and their default values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
<th>Default</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>Capacity of the production soil (SMA) store</td>
<td>mm</td>
<td>350</td>
<td>1-1500</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>Water exchange coefficient</td>
<td>mm</td>
<td>0</td>
<td>-10.0 to 5.0</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>Capacity of the routing store</td>
<td>mm</td>
<td>40</td>
<td>1-500</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>Time parameter for unit hydrographs</td>
<td>days</td>
<td>0.5</td>
<td>0.5-4.0</td>
</tr>
<tr>
<td>( k )</td>
<td>Filter parameter given by the recession constant (as in observed catchment runoff depth model)</td>
<td>none</td>
<td>n.a.</td>
<td>0-1</td>
</tr>
<tr>
<td>( C )</td>
<td>Shape parameter (as in observed catchment runoff depth model)</td>
<td>none</td>
<td>n.a.</td>
<td>0-1</td>
</tr>
</tbody>
</table>

Note that some coefficients in the model equations above appear as fixed values, eg. a power 4 in Equation 6 and Equation 20, a fixed split 10% - 90% of effective rainfall, a 2.5 exponent in the computation of the unit hydrographs, a 2.25 coefficient related to the percolation function in Equation 6. These values were chosen as those yielding the best model results in many different test conditions. They were fixed because leaving them free did not significantly improve (or even degraded) the model results while adding unhelpful complexity to the model structure.

Most optimisation algorithms used to calibrate the model parameter values require knowledge of an initial parameter set. This initial set may consist of median values obtained on a large variety of catchments (for example, see Table 3). Approximate 80% confidence intervals for the four parameters are also provided in Table 3. They were derived from the 0.1 and 0.9 percentiles of the distributions of model parameter values obtained over a large sample of catchments. Given the small number of model parameters, simple optimisation algorithms are generally capable of identifying parameter values yielding satisfactory results. The choice of an objective function depends on the objectives of model user. Note that care should be taken to set appropriate initial conditions of the internal state variables in the model to avoid discrepancies at the beginning of the simulation periods. One year can be used for model warm-up at the beginning of each simulation.

Table 3: Values of median model parameters and approximate 80% confidence intervals

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Median Value</th>
<th>80% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>350</td>
<td>100-1200</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0</td>
<td>-5 to 3</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>90</td>
<td>20-300</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>1.7</td>
<td>1.1-2.9</td>
</tr>
</tbody>
</table>

Leaving both the \( k \) and \( C \) parameters at 0 has the effect of disabling the baseflow filter, and all runoff will be reported as quickflow.
Output data

The model outputs daily surface flow and intercatchment groundwater exchange flow, expressed in mm/day.

The Source implementation of GR4J includes a baseflow filter component. This does not change the overall results of GR4J, but does introduce slow flow and quick flow as additional outputs, which can be useful in some circumstances. For example, the baseflow filter provides a means to use GR4J with the EMC/DWC constituent generation model. This baseflow filter is optional and there will be no effect on any results if the filter parameters ($k$ and $C$) are both left at their default values of 0. Use of this filter is described in the section on observed runoff, and also Hydrological Recipes, Estimation Techniques in Australian Hydrology, R. B. Grayson et al. 1996, Cooperative Research Centre for Catchment Hydrology, which can be downloaded at:

References


Bibliography


Edijatno, and C. Michel (1989), Un modèle pluie-débit journalier à trois paramètres, La Houille Blanche, 113-121.


