Piecewise Linear approach to Reservoir Routing - SRG

Storages are used to hold water for varying periods. They include dams and other reservoirs; weir pools; urban detention, retention or retarding basins; and natural lakes. In regulated river systems, storages control the supply of water to consumptive and non-consumptive users, and may also provide flood mitigation and environmental services. Typically, inflows to storages include stream flow from upstream catchments, rainfall over the storage surface area, recharge from groundwater, and runoff from the local catchment surrounding the storage. Outflows from storages include controlled releases and spills. Losses from storages include evaporation from the storage surface area and seepage to groundwater.

Controlled releases from a storage include discharge via regulated outlet structures such as gated spillways, valves, pumps and gates. The amount of water released is dependent upon downstream demands, storage operating rules and maximum and minimum release constraints. In river systems with ownership, releases are also influenced by owners' shares within the storage and the ownership of the outlet capacity.

Spills via gated spillways are modelled by specifying a minimum release for the gated spillway as a function of reservoir level. Pre-releases for flood control may be modelled using either the minimum release functionality of the gated spillway or a minimum flow node, for more complex pre-release rules such as seasonal targets.

Uncontrolled spills occur when a storage fills above the minimum level of an un-gated spillway, or the capacity of the gates on a gated spillway to control outflows is exceeded. Uncontrolled outflow may also occur through an uncontrolled outlet such as an ungated pipe culvert and via leakage through the dam wall.

The modelling of the physical operation of storages in Source is described below. Other functionalities related to storages are described in other SRG sections; these functionalities include:

- Resource assessment and water allocation
- Ownership
    - of storage volume, inputs, losses, spills and outlet capacities
    - Internal spilling between owners
    - Internal ceding (transfer from one owner to another based on agreed protocols)
    - Local borrow and payback systems.
- Weir operation (where the routing of flows in the headwater is significant - ie long flat weir pools)
- Ordering
    - Storages in series (orders passed upstream and transfer between storages),
    - Storages in parallel (choice of supply storage, optimisation of order delivery).
- Water quality in storages.

The ability to model the physical behaviour of storages is essential for fulfilling one of the primary purposes of Source, which is to model regulated river systems.

Scale

This node, in common with all others, is treated as a point location even though the storage represented may have large dimensions. It can therefore be considered to be site scale. It is used at every model time-step.

Principal developer

eWater CRC

Version

3.8.16

Scientific provenance

The principles of modelling the physical behaviour of storages have been presented in many text books over many years. Recent examples include books by Loucks and van Beek (2005), and McMahon and Adeloye (2005).

The approach adopted in Source uses numerical integration to solve the water balance equation for each model time-step, rather than the more usual finite difference approach. The advantage of numerical integration is that the technique is time-step independent whereas the finite difference technique will give different answers depending on the time-step used.

Version

Source version 2.16

Dependencies

A storage is represented as a node. It is connected to a single upstream link and can have multiple downstream links.

Structure and processes

Assumptions:

- The solution technique used assumes inflows, loss and gain fluxes, and outflows are averaged over a model time-step. This is consistent with the approach used in other parts of Source, such as streamflow routing;
- The storage reservoir is assumed to have a level pool; and
- Relationships between storage water level, volume, surface area, and outflows are defined in terms of piecewise linear, monotonically increasing coordinate sets.

Definitions

The following definitions supplement those in the eWater Glossary:

Owner

An entity that owns a share of water in a river system; not the same as a water user.

User

The person using the River Manager software.

Maximum Storage Level

The highest level at which water is contained within a storage. This level is the highest level defined in the storage dimensions.

Full Supply Level (FSL)

Spillway crest level for an uncontrolled spillway; for a controlled (gated) spillway, the maximum level for water supply storage based on operating and management decisions such as to maintain airspace for flood mitigation.

Dead Storage

The level/volume below which water cannot be released from the storage, that is equal to the lowest outlet invert level. This may differ from the inactive storage (or minimum operating) level, which may be defined at a higher level for water quality or power generation purposes

Flux

Any flow into or out of a storage, including inflow, controlled releases, spills, rainfall, evaporation, leakage and groundwater loss or gain.

Net Flux

The total flux combining all individual fluxes (inflow, rainfall, evaporation, groundwater flux and discharge).

m AHD

Levelin metres relative to the Australian Height Datum.

Minimum Release

The minimum rate of discharge from a storage when all outlets are closed and flow is only over spillway or through an uncontrolled outlet (including leakage).

Maximum Release

The rate of discharge from a storage when all outlets are fully open

Invert

The level of the base of the inside of a conduit (as distinct from the base of the outside, which includes the thickness of the casing)

PWL

Piecewise linear

Spill

water released in excess of the downstream requirements.

Theory

In summary, the adopted approach involves calculating the minimum and maximum discharge based on current inflows anduser defineddischarge, gain and loss relationships. Where there are multiple outlet paths, the minimum and maximum discharge is calculated for each path, assuming all paths are operational. If orders are less than the maximum discharge relationship, the order can be released. Otherwise, the order is constrained at the maximum discharge.

Source assumes that any flows (fluxes) into or out of the storage are averaged across the time-step. Flows and changes in storage volume are calculated by integrating across the time-step, assuming the reservoir has a level pool. The calculation process involves defining a series ofuser definedpiecewise linear relationships which describe storage dimensions, discharge relationships, and any losses and gains (such as evaporation and groundwater fluxes).

Calculation of storage mass balance

The calculation of storage mass balance during a model time-step is based on solving for the change in storage that occurs taking into account inflows, releases and all other loss and gain fluxes (eg net evaporation, seepage and spills).

Inflow, loss and gain fluxes, and outflows are assumed to be averaged over a model time-step. The combination of these fluxes describes the change in storage volume over a time-step. Fluxes and changes in storage volume are calculated by integrating across the time-step. This involves fulfilling the requirements outlined below.

Evaluation of net evaporation loss from storage

Inflow is generally estimated to the dam wall, and there is potential to double count water from the drowned section of the dam. This can be significant, especially where the reservoir area is a significant proportion of the catchment area. Hence, an adjustment is necessary to minimise the effect of double counting, and this is achieved by calculating the net evaporation loss. The net evaporation loss is the difference between the actual evaporation from the open water surface of the reservoir (*Eo*) and the actual evapotranspiration (Et) from the reservoir area assuming no dam (McMahon and Mein,1986: p136-137 & p185).

Linsley et al (1982) summarise the requirement for adjusting evaporation losses for the effect of the reservoir area as follows: "In reservoir design, the engineer is really concerned with the increased loss over the reservoir site resulting from the construction of the dam, ie. reservoir evaporation less evapotranspiration under natural conditions". This holds equally for modelling the behaviour of existing reservoirs. The mass balance calculations described below take this adjustment into account.

Model set up

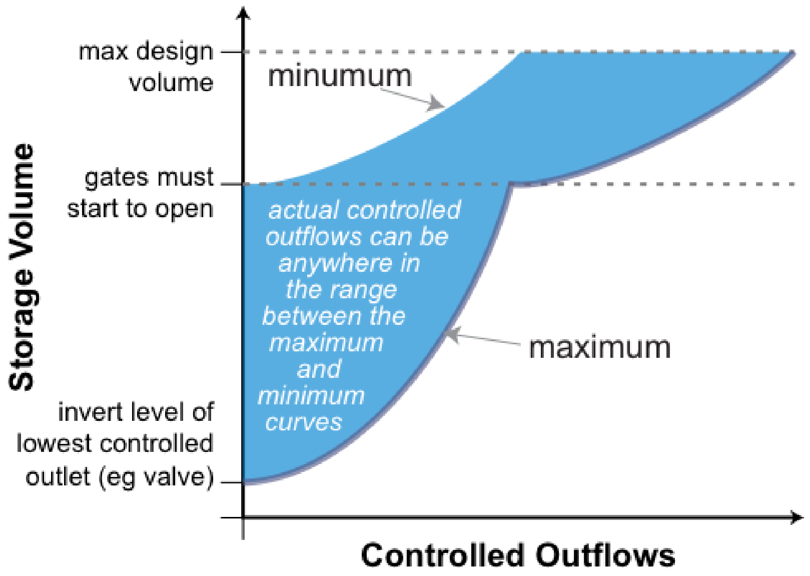
Model set up:

- Time series of open water evaporation, rainfall over the storage and runoff depths applicable to the storage area assuming this area is dry, are optionally read in. In the mass balance calculations, changes in storage volume due to these parameters are a function of storage surface area.
- The user must define piecewise linear relationships between storage volume and surface area and, optionally, water level (head). It should be noted, though, that the only place where water level is used in storage calculations is with hydropower and groundwater flux (it is also used in connection with wetlands).
- The user must also define piecewise linear relationships between storage volume, or water level, and maximum discharge through controlled outlets (ie. assuming the outlet is fully open), such as valves, where these are present or proposed; these relationships and the relationships between storage volume, surface area and water level can use different volume coordinates and the code interpolates additional points to create a common set of volume coordinates for all relationships.
- Gated spillways, where these are present or proposed, are treated as special cases of controlled outlets in that a piecewise linear relationship between storage volume, or water level, and minimum outflow is required as well as a piecewise linear relationship between storage volume, or water level, and maximum outflow. (Recalling that for other types of controlled outlet the minimum is zero.)
- Further piecewise linear relationships between storage volume, or water level, and discharge are required for unregulated outlets (eg. ungated spillways, ungated pipes or culverts). Examples of maximum and minimum controlled outflow relationships with either a gated spillway or an ungated spillway are shown in Figure 1 other combinations of outlets are possible.
- Each outlet type, whether controlled or uncontrolled, is assigned to an outlet path by the user. Each outlet path could have more than one outlet type assigned to it; for example, a valve and a gated spillway could both be assigned to the same outlet path. (Hence, if more than one type of uncontrolled outlet is assigned to a given outlet path then a combined piecewise linear relationship could be specified for these; the same applies to controlled outlets, but relationships for controlled and uncontrolled outlets should not be combined for reasons that will become apparent later on.)
- Groundwater flux (seepage when an outflow), expressed as a discharge, is a function of storage level and a piecewise linear relationship is required for this as well.
- Multiple outlets are handled by specifying more than one outlet path, but each outlet can only be assigned to one outlet path; that is, an outlet cannot be shared between outlet paths.

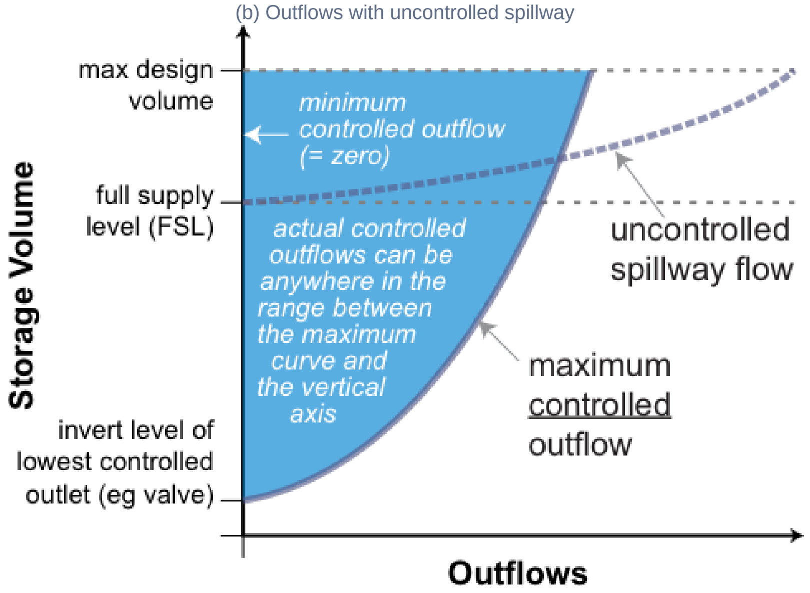
- Where there are multiple owners, these are also associated with outlet paths. Sharing of access to outlet capacity by owners can be based on the percentage of active volume or fixed shares (with or without sharing according to priority levels, for both options).

Figure 1. Examples of piecewise linear relationships between outflows and storage volume

(a) Controlled outflows with gated spillway



(b) Outflows with uncontrolled spillway



Flow phase

During the flow phase, the potential minimum and maximum discharges from a storage are calculated for a given model time-step based on current inflows, flux data and theuser definedpiecewise linear relationships. If orders on a given outlet are less than the maximum discharge, the order can be released. Otherwise, the order is constrained at the maximum allowable discharge. The steps involved in these calculations for a given model time-step are discussed in the next sections, but in summary they are as follows:

- Calculate the storage volume at the end of the time-step and minimum outflows during time-step, by integration, for which it is assumed all controlled outlets are closed (unless there are mandatory minimum outflow requirements when there is a gated spillway).
- Calculate the storage volume at the end of the time-step and maximum outflows during time-step, by integration, for which it is assumed all controlled outlets are fully open.
- Compare the maximum and minimum outflows during time-step with the water orders for the time-step. There are three possible outcomes:
    - If the water orders are less than the maximum but greater than the minimum then the orders can be met by releases.
    - If the water orders are greater than the maximum outflow then constrain the orders to the maximum that can be released.
    - If the water orders are less than the minimum outflow then the orders can be met through the uncontrolled outflows and no controlled releases are necessary. The minimum outflow occurs regardless of any water orders.
- If the water orders are less than the maximum but greater than the minimum (option (a) above) then recalculate the storage volume at the end of the time-step and outflows during time-step, by integration, based on water orders being able to be met by releases (Noting that in some circumstances, orders may be partly met by uncontrolled outflows, which assumes controlled outlets may have to be continually adjusted to achieve a constant release rate.).

As PWL relationships are used, each segment of a given PWL relationship may be expressed in terms of Equation 1:

Equation 1	$$Q = \frac{dv}{dt} = m \bullet v + c$$

where:

Q average flow per unit time

v volume

dt time

m, *c* slope and intercept of the segment, respectively

The mass balance equation is (Equation 2):

Equation 2	$$Net\ Flux = I - (E_o - P + R) - S - U - O$$

where:

I inflow

R runoff from storage area that would occur if this area were dry

P rainfall over the storage area in the current model time-step

E_o open water evaporation in the current model time-step

S groundwater flux (note a negative value of S means a gain to the storage; a positive value is equivalent to seepage)

U uncontrolled outflows

O controlled outflows

The runoff, rainfall and open water evaporation depth values read in as data are converted to volumes by multiplying them by the appropriate storage surface area. Also the term (E_o - P + R) is the net evaporation loss referred to in the section on evaluation of net evaporation loss from storage above.

Calculations for a given time-step where there is one outlet and one owner

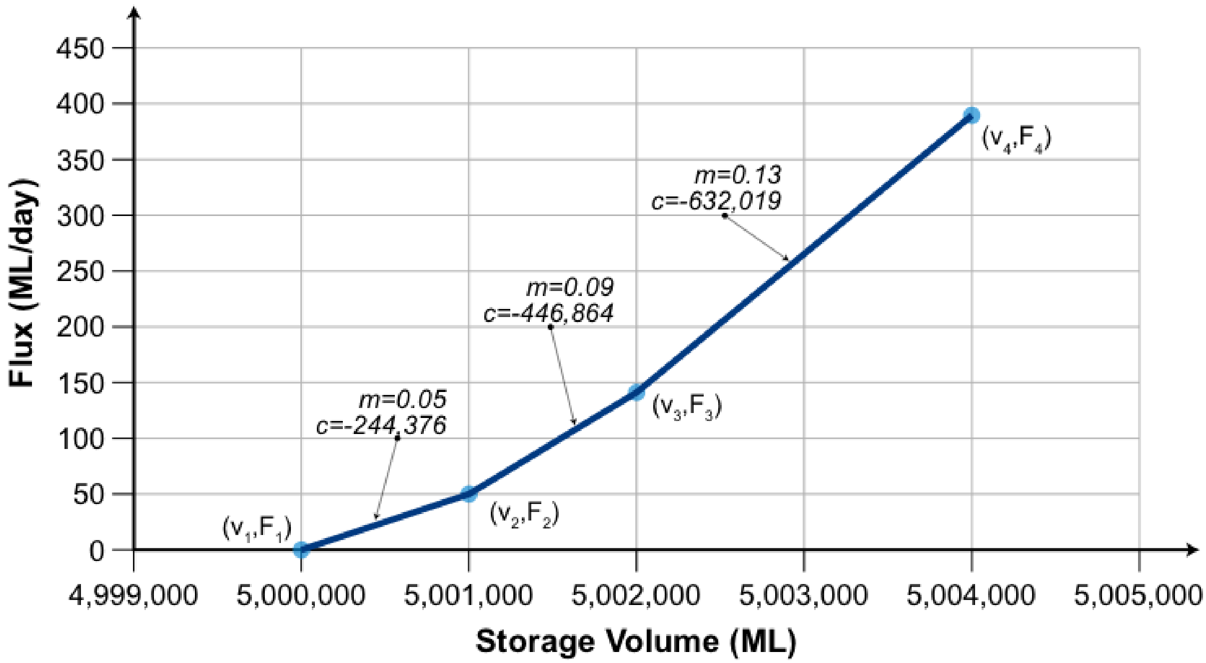
Step 1: Evaluate PWL relationships for the following fluxes:

- Open water evaporation
- Rainfall
- Runoff
- Groundwater
- Uncontrolled outflows
- Minimum controlled outflows
- Maximum controlled outflows

Interpolation is needed to bring all of these to a common set of storage volume, area and depth coordinates.

Step 2: Calculate the slope (*m*) and intercept (*c*) of each segment of the PWL relationships between storage volume and each individual flux derived in the previous step (see Figure 2) using Equations 3 and 4:

Figure 30. Example of a piecewise linear relationship between a flux and storage volume



Equation 3	$$m = \frac{F_2 - F_1}{v_2 - v_1}$$

Equation 4	$$c = F_1 - m \bullet v_1$$

where:

F_1, F_2 dependent variable (eg fluxes) at each end of a segment in the PWL relationship

v_1, v_2 equivalent storage volumes

Recall that the minimum controlled outflows are usually zero in all segments of the PWL relationship, except that when there is a gated spillway there may be mandatory minimum release requirements once the volume in the storage exceeds certain thresholds, as illustrated in Figure 1(a). Also, as inflows are constant, *m* = 0 and *c* = inflow value for inflows.

Step 3: By combining c and m values for individual fluxes by substituting into Equation 2, PWL relationships expressed in terms of *c* and *m* values are derived for:

- combined fluxes with minimum controlled outflows; and
- combined fluxes with maximum controlled outflows.

Step 4: To find the storage volume at the end of a time-step and outflows during the time-step, it is necessary to integrate Equation 1 with respect to time. The general form of the resultant equation is Equation 5:

Equation 5	$v = ke^{m \bullet t} - \frac{c}{m}$

where:

t time, as a fraction of the model time-step (ie. 0 *t* 1)

k a constant

Other terms as previously defined.

Equation 5 applies only when *m* 0, ie. when the net flux through the storage is variable across a segment of piecewise linear relationship. When *m* = 0, Equation 6:

Equation 6	$v = volume_{at\ start\ of\ time\ step} + Net\ Flux.dt$

The constant *k* can be calculated from Equation 5 by setting time *t* = 0 (start of time-step) and *v* = storage volume at start of time-step (v_0). Hence Equation 7:

Equation 7	$k = v_0 + \frac{c}{m}$

Step 5: The segment of the PWL relationships the volume at the start of the time-step lies in is established by searching for the case where:

$$VolumeCoordinate_{at\ lower\ limit\ of\ segment} < v_0 < VolumeCoordinate_{at\ upper\ limit\ of}$$

Step 6: For both minimum and maximum outflow cases, evaluation of the storage volume at the end of the time-step (v_t) and outflows during the time-step involves an iterative process in which the segment of the PWL relationships v_t lies in is established. Firstly, it is assumed the storage volume at the end of the time-step lies in the same segment of the PWL relationships as the storage volume at the start of the time-step, and v_t is estimated using Equation 5 with *t* = 1.

Step 7: The validity of the above assumption is then checked:

If:

$$VolumeCoordinate_{at\ lower\ limit\ of\ segment} < v_t < VolumeCoordinate_{at\ upper\ limit\ of}$$

then:

- the assumption is correct;
- the estimate of v_t is the required result;
- *t* = 1; and
- Step 9 is skipped.

Step 8: If not, then the required result lies in another segment of the PWL relationship (higher or lower), and it is necessary to establish the fraction of the time-step when the storage volume goes out of the current segment. This is calculated using Equation 8:

Equation 8	

$$t = \frac{1}{m} \bullet \ln\left(\frac{v_t + \frac{c}{m}}{k}\right)$$

$$t_e = t_e + t$$

where:

> t time that storage volume exits current segment, as a fraction of the model time-step
>
> t_e cumulative time since start of time-step that storage volume exits current segment, as a fraction of the model time-step (noting that initially t_e= 0)

Other terms as previously defined.

The remaining fraction of the time-step is $t_r = 1 - t_e$.

Step 9: Individual flux volumes (*Net Loss*, *S*, each component of *D*) for the current segment are determined using the value of *t* from step 7 or step 8 as appropriate, using Equation 9:

Equation 9	$v_F = \frac{m_F}{m_T} \bullet k \bullet e^{m_T \bullet t} - \frac{c_T \bullet m_F}{m_T} \bullet t + c_F \bullet t - \frac{m_F}{m_T} \bullet k$

where:

> t time
>
> v_F volume leaving or entering a storage due to a particular flux (initially v_F = 0)
>
> m_F slope of the linear relationship for the individual flux
>
> m_T slope of the linear relationship for the Net Flux
>
> c_F intercept of the linear relationship for the individual flux
>
> c_T intercept of the linear relationship for the Net Flux
>
> k constant defined using Equation 7 for the Net Flux

Note that when m_F = 0 or m_T = 0, Equation 10:

Equation 10	$v_F = v_F + c_F \bullet t$

Step 10: If *t* 1 then the cycle starting from step 4 through to step 9 is repeated for the new segment, using Equation 5 with $t = t_r$. A new value of *k* has to be calculated using Equation 7, still with *t* = 0. Values of *c* and *m* appropriate for the new segment, previously calculated with Equation 3 and Equation 4, are used.

Step 11: When *t* = 1 orders are compared with minimum and maximum release rates:

Equation 11	$OrderVolume = \min(OrderVolume, MaximumReleaseVolume)$

Equation 12	$ReleaseVolume = \max(OrderVolume, MinimumReleaseVolume)$

Step 12: If there are orders in this time-step and minimum release < orders < maximum release, steps 4 to 11 are repeated using constant outflows from step 11 (ie. *m* = 0 and *c* = outflow value) instead of the relationships that are a function of storage volume used previously.

Step 13: The final storage volume is recalculated by substituting the individual flux volumes into Equation 13:

Equation 13	$$StorageVolume_{Final} = StorageVolume_{initial} + I - (E_o - P + R) - S - U - O$$

where:

I, E_o, P, R, S, U and O are calculated individual flux volumes

The result is checked to ensure it is equal to the final result for v_t obtained in step 6 (ie the result from Equation 5 with t = 1).

Calculations for a given time-step where there is more than one outlet but only one owner.

This functionality is still under development as at September 2011 (Source version 2.16).

Calculations for a given time-step where thereismore than one outlet and more than one owner

This functionality is still under development as at September 2011 (Source version 2.16).

Multiple outlet paths

When a storage has more than one outlet path, the adaptive storage release method is automatically used by Source. Releases from storages with one outlet path are assumed to be constant through the time-step (limited by volume to the min/max release curves). The Adaptive Storage Release method generates a release curve based on the orders combined with the outlet curve. With this method, the storage releases at the maximum release rate where the storage cannot release at ordered rate. The storage releases at minimum release rate when it is greater than the order. The adaptive storage release method generates small artifacts when switching between the order and maximum/minimum release rates. However, provides better handling of releases when there are multiple outlets with big operating ranges.

Manual overriding of certain calculated storage parameters

For modelling forreal timeriver operations using River Operator, in Source, there is a 'warm-up' period where the model is run to ensure modelled fluxes equal observed values. Forecast releases are then used to predict the river response to a range of scenarios before the operator makes a decision on how much water to release or which path (multiple supply paths) to send water down. Source provides functionality which allows the user to manually override the simulated releases and storage volume/levels during the observed period. It also provides the ability to input forecast releases that override the model values.

Points to note include:

- When overriding releases, values used are still constrained to be in the range between the minimum and maximum release curves.
- If observed values are missing then modelled values are used.
- When simulated releases or storage volume/levels are overridden with different values, this may have an impact on various water user accounts, and these accounts are adjusted accordingly.
- When there are multiple owners, differences between simulated and input storage volumes (noting that if levels are input these are converted to equivalent volumes) are shared to owners' volumes according to evaporation and rainfall sharing rules.

Input data

Data requirements are discussed in the Theory section above. Full details on input data are provided in the Source User Guide.

Parameters and settings

Whileuser definedinput relationships are generally expressed in terms of storage levels, these are converted to volume relationships within the model. Alluser definedrelationships are entered as, or converted to, piecewise linear (PWL) relationships. All sets of values in the relationships should be monotonically increasing unless noted otherwise. The intervals within the PWL relationships should be sufficiently small that the assumption of linearity, and linear interpolation, are approximately correct.

Where the storage volume rises above the highest value specified in the PWL relationship, values are linearly extrapolated using the slope of the line for the last twouser definedco-ordinates. It is preferable for the user to define all PWL relationships to include extremes, as linear extrapolation may not appropriately represent the desired behaviour.

The PWL relationships must start at a zero volume or zero discharge, and are not extrapolated below the lowest values.

Calibration parameters

Storage parameters the user may want to use in calibration mode include:

- Storage releases for each outlet link (replace modelled releases with observed releases)
- Storage level/volume (replace the modelled storage volume with an observed storage volume)
- Storage ownership (replace modelled storage ownership with observed ownership levels - noting this is an accounting exercise so not really an observed value).

This uses the functionality for overriding calculated values discussed in the section on Manual overriding of certain calculated storage parameters above.

Output data

Outputs from the storage node include the following results:

- Inflow: flow and ownership
- Inflow Volume: volume and ownership
- Storage Level
- Storage Volume: total volume and ownership
- Dead Storage Volume: total volume and ownership
- Target Volume
- Surface Area
- Rainfall Volume: total volume and ownership
- Evaporation Volume: total volume and ownership
- Groundwater Flux Volume: total volume and ownership
- Maximum Release Rate
- Minimum Release Rate
- Order Volume - Link x: total volume and ownership
- Outflow
- Outflow Volume - Link x: total volume and ownership
- Spill Volume - Link x: total volume and ownership
- Controlled Release Volume - Link x: total volume and ownership
- Allocated Release Volume - Link x: total volume and ownership
- Unallocated Release Volume - Link x: total volume and ownership

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